P425/1
PURE MATHEMATICS
MAY 2025

**UGANDA ADVANCED CERTIFICATE OF EDUCATION**

**SET 10**

Paper 1

3 hours

**INSTRUCTIONS TO CANDIDATES:**

- Answer all the **eight** questions in section **A** and only **five** questions in section **B**.
- Any additional question(s) will not be marked.
- **All** working **must** be shown clearly.
- Graph paper is provided.
- Silent non-programmable scientific calculators and mathematical tables with a list of formulae may be used.
- Clearly indicate the questions you have attempted on the answer scripts as illustrated, **DO NOT** hand in the question paper.

## SECTION A (40 MARKS)

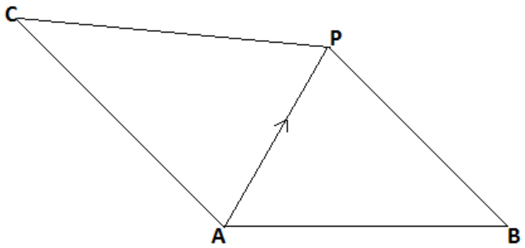
1. Given that $a = \log_5 35$ and $b = \log_9 35$, show that $\log_5 21 = \frac{1}{2b}(2ab - 2b + a)$.
(5 marks)

2. Solve the inequality: $|x-2| > 3|2x+1|$. (5 marks)

3. Prove that: $\tan^{-1}\frac{1}{2} - \operatorname{cosec}^{-1}\frac{\sqrt{5}}{2} = \cos^{-1}\frac{4}{5}$. (5 marks)

4. Expand $\frac{1}{\sqrt{1+x}}$ up to the term in $x^2$ and by letting $x = \frac{1}{4}$, show that $\sqrt{5} \approx \frac{256}{115}$. (5 marks)

5.




$A$, $B$, $C$ and $P$ are four points such that $3\overrightarrow{\mathbf{AP}} = 2\overrightarrow{\mathbf{AB}} + \overrightarrow{\mathbf{AC}}$, show that $B$, $P$ and $C$ are collinear and that $P$ is the point of trisection of the line $BC$. (5 marks)

6. Given that $y = \frac{e^x - e^{-x}}{e^x + e^{-x}}$, show that $\frac{d^2y}{dx^2} + 2y\frac{dy}{dx} = 0$. (5 marks)

7. Find the volume of the solid generated by rotating the area bounded by the curve $y = \cos\frac{1}{2}x$ from $x = 0$ to $x = \pi$ about the $x-$ axis.
(5 marks)

8. Solve the d.e given $\cos x\frac{dy}{dx} - 2y\sin x = 1$. (5 marks)

## SECTION B: (60 MARKS)

9a) Evaluate the coefficient of $x$ in the expansion of $\left(x+\frac{2}{x^2}\right)^{10}$. (5marks)

b) Prove by Mathematical induction that: $\sum_{r=2}^{n}\frac{1}{r^2-1}=\frac{3}{4}-\frac{2n+1}{2n(n+1)}$.

(7 marks)

10a) Prove the identity: $\cos^6 x+\sin^6 x=1-\frac{3}{4}\sin^2 2x$. (6marks)

b) Solve the equation: $4\sin^2 x+8\cot^2 x=5\operatorname{cosec}^2 x$ for $0\le x\le 2\pi$. (6marks)

11. Sketch the curve $y=\frac{4+3x-x^2}{x-8}$, clearly find the nature of the turning points and state their asymptotes. (12marks)

12a) The points $P\left(5p,\ \frac{5}{p}\right)$ and $Q\left(5q,\ \frac{5}{q}\right)$ lie on the rectangular hyperbola $xy=25$. Find the equation of the tangent at $P$ and hence deduce the equation of the tangent at $Q$.

b) The tangents at $P$ and $Q$ meet at point $N$, show that the coordinates of $N$ are $\left(\frac{10pq}{p+q},\ \frac{10}{p+q}\right)$, hence find the locus of $N$ given that $pq=-1$.

(12marks)

13a) Given that $z(5+5i)=a(1+3i)+b(2-i)$ where $a$ and $b$ are real numbers and that $\arg z=\frac{\pi}{2}$ and $|z|=7$, find the values of $a$ and $b$. (6marks)

b) Describe the locus of the complex number $z$ when it moves in the argand diagram such that $\arg\left(\frac{z-3}{z-2i}\right)=\frac{\pi}{4}$. (6 marks)

14a) Evaluate: $\int_0^{\pi/3} x\sin 3x\, dx$ (6 marks)

b) Prove that: $\int_{\pi}^{4\pi/3} \operatorname{cosec}\frac{1}{2}x\, dx = In3$ (6 marks)

15a) Find the point of intersection between the plane $\mathbf{r}\,.\left(2\mathbf{i}-\mathbf{j}+\mathbf{k}\right)=\mathbf{4}$ and the line passing through the point $(3,\ 1,\ 2)$ and is perpendicular to this plane.

(5 marks)

b) Find the perpendicular distance of the point $(4,\ -3,\ 10)$ to the line

$\frac{x-1}{3} = 2-y = \frac{z-3}{2}$. (7 marks)

16. A liquid is being heated in an oven maintained at a constant temperature of $180^oC$. It is assumed that the rate of increase in the temperature of the liquid is proportional to $(180-\theta)$, where $\theta^\circ C$ is the temperature of the liquid at time $t$ minutes. If the temperature of the liquid rises from $0^oC$ to $120^oC$ in 5 minutes, find the temperature of the liquid after a further 5 minutes. (12marks)

## SET 10 MARKING GUIDE 2025

| | | |
|---|---|---|
| 1. | $a=\log_5 35=\log_5(5\times 7)=1+\log_5 7$, $\log_5 7=a-1$<br>$b=\log_9 35=\frac{\log_5 35}{\log_5 9}=\frac{a}{2\log_5 3}$, $\log_5 3=\frac{a}{2b}$<br>$\log_5 21=\log_5 7+\log_5 3=a-1+\frac{a}{2b}=\frac{1}{2b}(2ab-2b+a)$ | |
| 2. | $(x-2)^2>9(2x+1)^2$, $x^2-4x+4>9(4x^2+4x+1)$, $7x^2+8x+1<0$<br>$(7x+1)(x+1)<0$ | |

| | $x<-1$ | $-1<x<-\frac{1}{7}$ | $x>-\frac{1}{7}$ |
|---|---|---|---|
| $(7x+1)$ | $-$ | $-$ | $+$ |
| $(x+1)$ | $-$ | $+$ | $+$ |
| *sign* | $+$ | $-$ | $+$ |

solution

$-1<x<-\frac{1}{7}$

3. Let $\tan^{-1}\frac{1}{2}=A$, $\operatorname{cosec}^{-1}\frac{\sqrt{5}}{2}=B$ thus $\tan A=\frac{1}{2}$ and $\operatorname{cosec}B=\frac{\sqrt{5}}{2}$,

$\sin A=\frac{1}{\sqrt{5}}$, $\cos A=\frac{2}{\sqrt{5}}$, $\sin B=\frac{2}{\sqrt{5}}$, $\cos B=\frac{1}{\sqrt{5}}$

$A-B=\cos^{-1}(\cos(A-B))$

$A-B=\cos^{-1}(\cos A\cos B+\sin A\sin B)$

$A-B=\cos^{-1}\left(\frac{2}{\sqrt{5}}\times\frac{1}{\sqrt{5}}+\frac{1}{\sqrt{5}}\times\frac{2}{\sqrt{5}}\right)$ $\quad A-B=\cos^{-1}\left(\frac{4}{5}\right)$ as required.

4.

$$\frac{1}{\sqrt{1+x}}=(1+x)^{-\frac{1}{2}}=\left\{1+\left(-\frac{1}{2}\right)x+\frac{\left(-\frac{1}{2}\right)\left(-\frac{1}{2}-1\right)}{2!}x^2+\ldots\right\}$$

$$=1-\frac{x}{2}+\frac{3}{8}x^2+\ldots$$

For $x=\frac{1}{4}$, $\frac{1}{\sqrt{1+\frac{1}{4}}}\approx 1-\left(\frac{1}{2}\right)\left(\frac{1}{4}\right)+\left(\frac{3}{8}\right)\left(\frac{1}{4}\right)^2$

$\therefore \sqrt{\frac{4}{5}}\approx\frac{115}{128}$ , $\frac{2}{\sqrt{5}}\approx\frac{115}{128}$ , so $\sqrt{5}\approx\frac{256}{115}$

5. $\overrightarrow{\mathbf{AP}}=\overrightarrow{\mathbf{AB}}+\overrightarrow{\mathbf{BP}}$ and $\overrightarrow{\mathbf{AP}}=\overrightarrow{\mathbf{AC}}+\overrightarrow{\mathbf{CP}}$

But $2\overrightarrow{\mathbf{AP}}=2\overrightarrow{\mathbf{AB}}+2\overrightarrow{\mathbf{BP}}$ ...$(i)$ and $\overrightarrow{\mathbf{AP}}=\overrightarrow{\mathbf{AC}}+\overrightarrow{\mathbf{CP}}$ ...$(ii)$

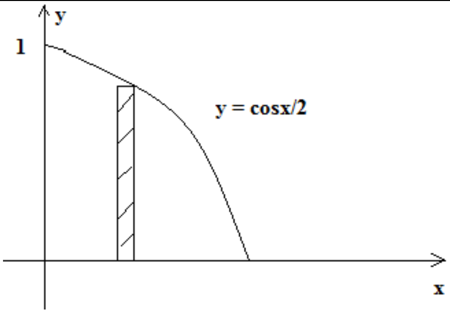
So eqn(i) + eqn(ii), $3\overrightarrow{\mathbf{AP}}=2\overrightarrow{\mathbf{AB}}+2\overrightarrow{\mathbf{BP}}+\overrightarrow{\mathbf{AC}}+\overrightarrow{\mathbf{CP}}$

$\therefore$ $2\overrightarrow{\mathbf{BP}}=-\overrightarrow{\mathbf{CP}}=\overrightarrow{\mathbf{PC}}$ thus, $\overrightarrow{\mathbf{BP}}$ is parallel to $\overrightarrow{\mathbf{PC}}$ and $P$ is the common point so $B$, $P$ and $C$ are collinear and $P$ is the point of trisection of the line $BC$.

6. $\frac{dy}{dx}=\frac{(e^x+e^{-x})(e^x+e^{-x})-(e^x-e^{-x})(e^x-e^{-x})}{(e^x+e^{-x})^2}$, $\frac{dy}{dx}=\frac{(e^x+e^{-x})^2}{(e^x+e^{-x})^2}-\frac{(e^x-e^{-x})^2}{(e^x+e^{-x})^2}$

$\frac{dy}{dx}=1-y^2$, $\frac{d^2y}{dx^2}=-2y\frac{dy}{dx}$, so $\frac{d^2y}{dx^2}+2y\frac{dy}{dx}=0$

7.




$$V=\pi\int_0^{\pi}\cos^2\frac{1}{2}x\,dx$$

$$V=\frac{\pi}{2}\int_0^{\pi}(1+\cos x)\,dx \qquad V=\frac{\pi}{2}[x+\sin x]_0^{\pi}$$

$$V=\frac{\pi}{2}((\pi+\sin\pi)-(0)) \qquad V=\frac{\pi^2}{2}\,cubic\,units$$

8.

$$\cos x\frac{dy}{dx}-2y\sin x=1$$

$$\frac{dy}{dx}-2y\tan x=\sec x$$

$$I.F\;\;R=e^{-2\int\tan x\,dx}=e^{2In\cos x}=\cos^2 x$$

So, $\frac{d}{dx}\left(y\cos^2 x\right)=\cos^2 x\,.\frac{1}{\cos x}$ thus $d\left(y\cos^2 x\right)=\cos x\,.dx$

$$y\cos^2 x=\sin x+c\,,$$

thus $y=\tan x\sec x+c\sec^2 x$

9a)

${}^{10}C_r(x)^{10-r}\left(\frac{2}{x^2}\right)^r=Ax\,,\quad A={}^{10}C_r\,2^r\,,\quad$ and $(x)^{10-r-2r}=x^1$

$\Rightarrow 10-3r=1$, thus $r=3$

$$A={}^{10}C_3 2^3=\frac{10\times9\times8\times7!}{7!\times3\times2\times1}\times2^3=960$$

b)

$$\frac{1}{3}+\frac{1}{8}+\frac{1}{15}+\ldots+\frac{1}{n^2-1}=\frac{3}{4}-\frac{2n+1}{2n(n+1)}$$

$$\frac{1}{3}+\frac{1}{8}+\frac{1}{15}+\ldots+\frac{1}{n^2-1}=\frac{3}{4}-\frac{2n+1}{2n(n+1)}$$

For $n=2$, L.H.S = $\frac{1}{3}=\frac{3}{4}-\frac{5}{12}=\frac{1}{3}=$ R.H.S  For $n=3$,

L.H.S = $\frac{11}{24}=\frac{3}{4}-\frac{7}{24}=\frac{11}{24}$ = R.H.S

For $n=k$, we have $\frac{1}{3}+\frac{1}{8}+\frac{1}{15}+\ldots+\frac{1}{k^2-1}=\frac{3}{4}-\frac{2k+1}{2k(k+1)}$

For $n=k+1$, the L.H.S is

$$\frac{1}{3}+\frac{1}{8}+\frac{1}{15}+\ldots+\frac{1}{k^2-1}+\frac{1}{(k+1)^2-1}$$

$$=\frac{3}{4}-\frac{2k+1}{2k(k+1)}+\frac{1}{(k+1)^2-1}$$

| | | |
|---|---|---|
| | $= \frac{3}{4} - \frac{2k+1}{2k(k+1)} + \frac{1}{k(k+2)}$<br>$= \frac{3}{4} - \frac{1}{k}\left[\frac{2k^2+5k+2-2k-2}{2(k+1)(k+2)}\right]$<br>$= \frac{3}{4} - \frac{1}{k}\left[\frac{2k^2+3k}{2(k+1)(k+2)}\right]$<br>$= \frac{3}{4} - \left[\frac{2k+3}{2(k+1)(k+2)}\right]$ which is true for $n = k+1$ for the R.H.S, therefore if its true for $n = 2, 3, \ldots, n = k, n = k+1$ then the proof holds for all $n \geq 2$. | |
| 10a) | $\cos^6 x + \sin^6 x = (\cos^2 x)^3 + (\sin^2 x)^3$<br>$= (\cos^2 x + \sin^2 x)(\cos^4 x - \cos^2 x\sin^2 x + \sin^4 x)$<br>$= (\cos^4 x + \sin^4 x - \cos^2 x\sin^2 x)$<br>$= ((\cos^2 x + \sin^2 x)^2 - 2\cos^2 x\sin^2 x - \cos^2 x\sin^2 x)$<br>$= (1 - 3\cos^2 x\sin^2 x)$<br>$= 1 - 3\left(\frac{1}{2}\sin 2x\right)^2 \qquad = 1 - \frac{3}{4}\sin^2 2x$ | |
| b) | $4\sin^2 x + \frac{8\cos^2 x}{\sin^2 x} = \frac{5}{\sin^2 x}$<br>$4\sin^4 x + 8(1-\sin^2 x) = 5$, $4\sin^4 x - 8\sin^2 x + 3 = 0$,<br>$4\sin^4 x - 2\sin^2 x - 6\sin^2 x + 3 = 0$<br>$2\sin^2 x(2\sin^2 x - 1) - 3(2\sin^2 x - 1) = 0$<br>$(2\sin^2 x - 3)(2\sin^2 x - 1) = 0$<br>$\sin^2 x = \frac{3}{2}\ (Discarded)$<br>$\sin^2 x = \frac{1}{2}$, $\sin x = \pm\frac{1}{\sqrt{2}}$, $x = \frac{\pi}{4}, \frac{3\pi}{4}, \frac{4\pi}{3}, \frac{7\pi}{4}$ | |
| 11. | $\begin{array}{r} -x-5 \\ x-8\overline{)-x^2+3x+4} \\ -x^2+8x \\ -5x+4 \\ -5x+40 \\ -36 \end{array}$<br>$y = \frac{4+3x-x^2}{x-8}$, $\quad y = -x-5+\frac{-36}{x-8}$<br>Intercepts: $x = 0$, $y = -\frac{1}{2}$ so $\left(0, -\frac{1}{2}\right)$<br>$y = 0$, $4+3x-x^2 = 0$, $x^2-3x-4 = 0$, $(x-4)(x+1) = 0$, $x = 4$, $x = -1$,<br>so, $(4, 0)$ and $(-1, 0)$.<br>Vertical asymptotes: As $y \to \pm\infty$, $x \to 8$, so $x = 8$<br>As $x \to \pm\infty$, $y \to -x-5$, so $y = -x-5$ is the slanting asymptote. | |

$y = -x - 5 + -36(x-8)^{-1}$.

$\frac{dy}{dx} = -1 + 36(x-8)^{-2} = -1 + \frac{36}{(x-8)^2}$ for t.p $-1 + \frac{36}{(x-8)^2} = 0$

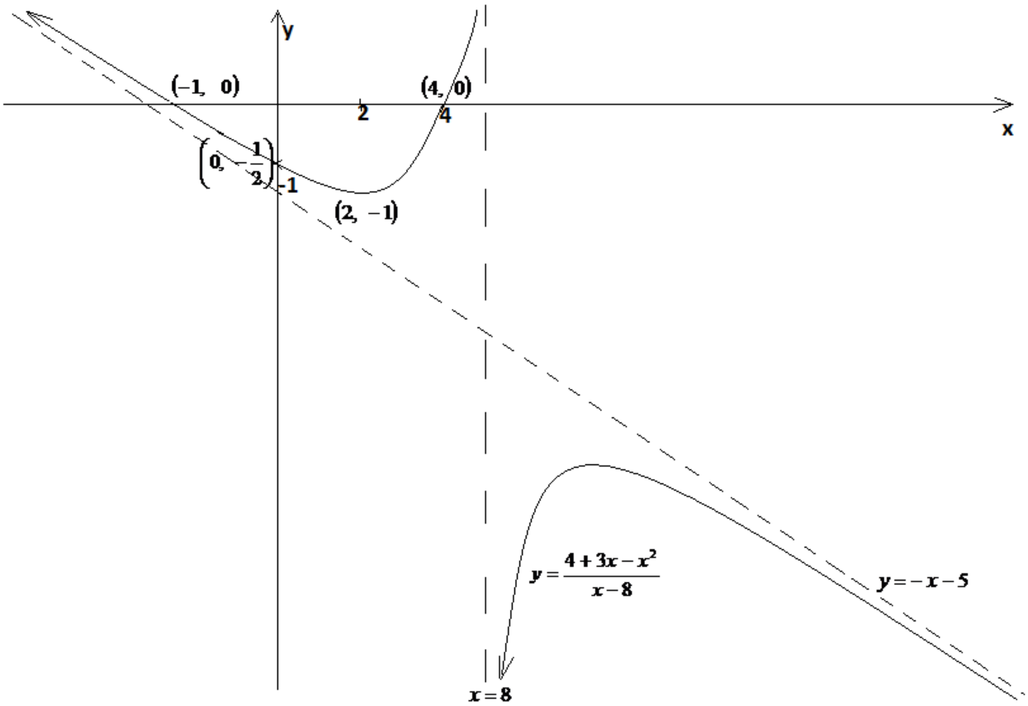
$(x-8)^2 = 36$, $(x-8) = \pm 6$, $x = 14$, $x = 2$

$x = 2$, $y = -1$ so $(2, -1)$min, $x = 14$, $y = -25$ so $(14, -25)$max

| | **L** | $x=2$ | **R** | **L** | $x=14$ | **R** |
|---|---|---|---|---|---|---|
| **Sign of** $\frac{dy}{dx}$ | $-$ | | $+$ | $+$ | | $-$ |

Region table

| | $x < -1$ | $-1 < x < 4$ | $4 < x < 8$ | $x > 8$ |
|---|---|---|---|---|
| $4 + 3x - x^2$ | $-$ | $+$ | $-$ | $-$ |
| $x - 1$ | $-$ | $-$ | $-$ | $+$ |
| $y$ | $+$ | $-$ | $+$ | $-$ |




**12a)** $y = \frac{25}{x}$, $\frac{dy}{dx} = -\frac{25}{x^2}$ at $P\left(5p, \frac{5}{p}\right)$, $\frac{dy}{dx} = -\frac{25}{25p^2} = -\frac{1}{p^2}$

Equation of the tangent is $\frac{y - \frac{5}{p}}{x - 5p} = -\frac{1}{p^2}$

$p^2 y - 5p = -x + 5p$, $x + p^2 y = 10p$ hence equation of tangent at $Q$ is $x + q^2 y = 10q$ .

**b)** $x + p^2 y = 10p$ …(i) $x + q^2 y = 10q$ …(ii)

$(p^2 - q^2)y = 10(p-q)$, $y = \frac{10(p-q)}{(p+q)(p-q)} = \frac{10}{(p+q)}$

| | | |
|---|---|---|
| | $x = 10p - \dfrac{10p^2}{(p+q)} = \dfrac{10p^2 + 10pq - 10p^2}{(p+q)} = \dfrac{10pq}{(p+q)}$ <br> Thus $N\left(\dfrac{10pq}{p+q}, \dfrac{10}{p+q}\right)$ <br> $\dfrac{x}{y} = pq = -1$ , thus locus is $x + y = 0$ | |
| 13a) | $z = 7\left(\cos\dfrac{\pi}{2} + i\sin\dfrac{\pi}{2}\right) = 7i$ <br> $7i(5+5i) = a(1+3i) + b(2-i)$ <br> $-35 + 35i = (a+2b) + i(3a-b)$ <br> $a + 2b = -35\ldots$(i) $3a - b = 35\ldots$(ii) <br> $6a - 2b = 70$ $a = 5$ and $b = -20$ | |
| b) | $\arg\left(\dfrac{z-3}{z-2i}\right) = \dfrac{\pi}{4}$. $\arg(z-3) - \arg(z-2i) = \dfrac{\pi}{4}$, for $z = x + iy$, <br> $\arg((x-3) + iy) - \arg(x + i(y-2)) = \dfrac{\pi}{4}$ <br> $\tan^{-1}\left(\dfrac{y}{x-3}\right) - \tan^{-1}\left(\dfrac{y-2}{x}\right) = \dfrac{\pi}{4}$, let <br> $\tan^{-1}\left(\dfrac{y}{x-3}\right) = A$, $\tan^{-1}\left(\dfrac{y-2}{x}\right) = B$ <br> $\tan(A-B) = \tan\dfrac{\pi}{4} = 1$, thus $\dfrac{\dfrac{y}{x-3} - \dfrac{y-2}{x}}{1 + \left(\dfrac{y}{x-3}\right)\left(\dfrac{y-2}{x}\right)} = 1$ <br> $\dfrac{xy - xy + 2x + 3y - 6}{x(x-3)} = \dfrac{x^2 - 3x + y^2 - 2y}{x(x-3)}$, <br> $x^2 + y^2 - 5x - 5y + 6 = 0$ <br> $(x - \frac{5}{2})^2 + (y - \frac{5}{2})^2 = \frac{26}{4}$, thus the locus is a circle with the centre $\left(\dfrac{5}{2}, \dfrac{5}{2}\right)$ <br> and radius $\dfrac{\sqrt{26}}{2}$ units. | |
| 14a) | $\int_0^{\pi/3} x\sin 3x\, dx$ $u = x$, $\frac{dv}{dx} = \sin 3x$ <br> $\frac{du}{dx} = 1$, $v = -\frac{1}{3}\cos 3x$ <br> $\int_0^{\pi/3} x\sin 3x\, dx = \left[-\dfrac{x}{3}\cos 3x\right]_0^{\pi/3} - \int_0^{\pi/3} -\dfrac{1}{3}\cos 3x\, dx$ <br> $= \left[-\dfrac{x}{3}\cos 3x\right]_0^{\pi/3} - \left[\dfrac{1}{9}\sin 3x\right]_0^{\pi/3}$ <br> $= -\left(\dfrac{\pi}{9}\cos\pi - 0\right) - \left(\dfrac{1}{9}\sin\pi - \dfrac{1}{9}\sin 0\right) = \dfrac{\pi}{9}$ | |

| | | |
|---|---|---|
| b) | $\int_{\pi}^{4\pi/3} \frac{1}{\sin x/2}\,dx$ Let $\sin\frac{x}{2} = \frac{2t}{1+t^2}$, $t = \tan\frac{1}{4}x$, $dt = \frac{1}{4}\sec^2\frac{1}{4}x dx$<br>$\int_{1}^{\sqrt{3}} \frac{1}{\left(\frac{2t}{1+t^2}\right)} \cdot \frac{4dt}{1+t^2}$ $x = \frac{4\pi}{3}$, $t = 1$<br>$x = \pi$, $t = \sqrt{3}$<br>$2\int_{1}^{\sqrt{3}} \frac{1}{t}dt = 2[Int]_{1}^{\sqrt{3}}$ $2\left(In\sqrt{3} - In1\right) = In3$ | |
| 15a) | Cartesian equation of plane is $2x - y + z = 4$<br>Parametric equations are, $x = 3 + 2t$, $y = 1 - t$, $z = 2 + t$<br>Thus $2(3+2t) - (1-t) + (2+t) = 4$, $6 + 4t - 1 + t + 2 + t = 4$, $6t = -3$,<br>$t = -\frac{1}{2}$<br>$x = 2$, $y = \frac{3}{2}$, $z = \frac{3}{2}$ so point is $\left(2,\ \frac{3}{2},\ \frac{3}{2}\right)$ or $2\mathbf{i} + \frac{3}{2}\mathbf{j} + \frac{3}{2}\mathbf{k}$ | |
| b) | If $N(x,\ y,\ z)$ is the foot of the perpendicular from the point<br>$P(4,\ -3,\ 10)$ to the line $\frac{x-1}{3} = 2 - y = \frac{z-3}{2}$, then the parametric<br>equation of $N$: $x = 1 + 3k$, $y = 2 - k$, $z = 3 + 2k$<br>The direction vector $\mathbf{d} = \begin{pmatrix} 3 \\ -1 \\ 2 \end{pmatrix}$, $\overrightarrow{\mathbf{PN}} \cdot \mathbf{d} = \mathbf{0}$<br>$\begin{pmatrix} x-4 \\ y+3 \\ z-10 \end{pmatrix} \cdot \begin{pmatrix} 3 \\ -1 \\ 2 \end{pmatrix} = 0$, $3x - 12 - y - 3 + 2z - 20 = 0$, $3x - y + 2z = 35$<br>$3(1+3k) - (2-k) + 2(3+2k) = 35$, $3 + 9k - 2 + k + 6 + 4k = 35$, $k = 2$<br>So $x = 7$, $y = 0$, $z = 7$, $\overrightarrow{\mathbf{PN}} = \begin{pmatrix} 3 \\ 3 \\ -3 \end{pmatrix}$, thus,<br>$\left\|\overrightarrow{\mathbf{PN}}\right\| = \sqrt{3^2 + 3^2 + (-3)^2} = 3\sqrt{3}\ units$<br>ALT<br>Given the point $P(4,\ -3,\ 10)$ and $A(1,\ 2,\ 3)$ is a point on the line, the<br>direction vector $\mathbf{d} = \begin{pmatrix} 3 \\ -1 \\ 2 \end{pmatrix}$<br>Distance $\mathbf{D} = \left\| \frac{\overrightarrow{\mathbf{AP}} \times \mathbf{d}}{\|\mathbf{d}\|} \right\|$,<br>$\overrightarrow{\mathbf{AP}} \times \mathbf{d} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 3 & -5 & 7 \\ 3 & -1 & 2 \end{vmatrix} = (-10+7)\mathbf{i} - (6-21)\mathbf{j} + (-3+15)\mathbf{k}$ | |

| | | |
|---|---|---|
| | $\overrightarrow{\mathbf{AP}} \times \mathbf{d} = -3\mathbf{i} + 15\mathbf{j} + 12\mathbf{k}$ , $\left\|\overrightarrow{\mathbf{AP}} \times \mathbf{d}\right\| = \sqrt{(-3)^2 + 15^2 + 12^2} = 3\sqrt{42}$ <br> $\mathbf{D} = \left\|\frac{3\sqrt{42}}{\sqrt{14}}\right\| = 3\sqrt{3}\ units$ | |
| 16. | The rate of increase in the temperature of the liquid is $\frac{d\theta}{dt}$ <br> $\therefore\ \frac{d\theta}{dt} = k(180-\theta)$, where $k$ is constant <br> Re-arranging, $\int \frac{1}{180-\theta} d\theta = \int k dt \quad \Leftrightarrow$ <br> $\int \frac{-1}{180-\theta} d\theta = -\int k dt$ <br> Thus for $\theta < 180$, $In(180-\theta) = -kt + c$ <br> $\therefore \quad 180-\theta = e^{-kt+c}$ writing $e^c = A$ <br> we get $\theta = 180 - Ae^{-kt}$ <br> let $t = 0$ when $\theta = 0$; thus $0 = 180 - A$ <br> $\therefore \quad \theta = 180(1 - e^{-kt})$ <br> Since $\theta = 120$ when $t = 5$, then $120 = 180(1 - e^{-5k})$ <br> i.e $e^{-5k} = \frac{1}{3}$ <br> Hence when $t = 10$, $\theta = 180(1 - e^{-10k})$ <br> $= 180(1 - \{e^{-5k}\}^2)$ <br> $= 180(1 - \{\frac{1}{3}\}^2) = 160$ <br> Thus the temperature of the liquid after a further 5 minutes is 160°C. | |